

11

Solutions

I. Concentration Units

A. Mass percent

1. Formula for determining mass percent:

$$\text{mass \%} = \frac{\text{mass part}}{\text{mass whole}} \times 100$$

2. Exercises

a. Calculate the mass percent of water in a solution made up of 123 g of NaOH and 289 g of water.

Solution: Since

$$\text{mass percent of water} = \frac{\text{mass of water}}{\text{mass of solution}} \times 100$$

we get

$$\text{mass percent of water} = \frac{289 \text{ g}}{123 \text{ g} + 289 \text{ g}} \times 100 = 70.1 \%$$

b. An aqueous solution of potassium iodide is often made up by mixing KI and I₂ and dissolving the mixture in water. A 5.06% KI solution has 18 g of KI and I₂ and 35 g of water. How much KI is in the mixture? (E1)

$$\% \text{ KI} = \frac{\text{mass KI}}{\text{mass solution}} \times 100$$

$$5.06\% = \frac{\text{mass KI}}{53 \text{ g}} \times 100$$

$$\text{mass KI} = \frac{2.68 \text{ g KI}}{15.32 \text{ g I}_2}$$

B. Parts per million (ppm)

1. Formula for determining parts per million:

$$\text{ppm} = \frac{\text{mass part}}{\text{mass solution}} \times 10^6 = (\text{mass } \%) (\times 10^4)$$

2. Mathematical relation between parts per million and mass percent:

$$\text{ppm} = (\text{mass percent}) \times 10^4$$

3. Exercises

a. Methyl mercury(II) chloride, CH_3HgCl , is a toxic pollutant usually found in waters around industrial sites. If a sample of 250.0 mL of water ($d = 1.00 \text{ g/mL}$) is found to contain 0.200 mg of CH_3HgCl , what is the concentration of methyl mercury(II) chloride in parts per million?

Solution: The formula we use is

$$\text{ppm} = \frac{\text{mass of solute}}{\text{total mass of solution}} \times 10^6$$

The mass of the solute, CH_3HgCl , is 0.200 mg, which is $2.00 \times 10^{-4} \text{ g}$. The mass of the solution should also be in grams. Since we are given a volume and density, we obtain

$$250.0 \text{ mL of solution} \times \frac{1.00 \text{ g solution}}{1.00 \text{ mL solution}} = 250.0 \text{ g solution}$$

Substituting now into the formula for ppm, we get

$$\text{ppm} = \frac{2.00 \times 10^{-4} \text{ g}}{250.0 \text{ g}} \times 10^6 = 0.800$$

b. A water solution contains 0.56 ppm of lead as a pollutant. If you drank a cup (1/2 pint) of this water, how many mg of lead would you ingest? (density of water = 1.00 g/mL) (E2)

$$1 \text{ cup} \left(\frac{0.5 \cancel{\text{ pt}}}{1 \cancel{\text{ cup}}} \right) \left(\frac{1 \cancel{\text{ qt}}}{2 \cancel{\text{ pt}}} \right) \left(\frac{1 \cancel{\text{ gal}}}{4 \cancel{\text{ qt}}} \right) \left(\frac{3.78 \cancel{\text{ L}}}{1 \cancel{\text{ gal}}} \right) \left(\frac{1000 \cancel{\text{ mL}}}{1 \cancel{\text{ L}}} \right) \left(\frac{1 \text{ g}}{1 \cancel{\text{ mL}}} \right) = 236.25$$

$$0.56 \text{ ppm Pb} = \frac{\text{mass Pb}}{236.25 \text{ g H}_2\text{O}} \times 10^6$$

$$\text{mass Pb} = 1.32 \times 10^{-4} \text{ g} \left(\frac{1000 \text{ mg}}{1 \text{ g}} \right) = \boxed{0.132 \text{ mg Pb}}$$

C. Mole fraction

Recall from Chapter 5 that the mole fraction of a component in a mixture is given by the relation

$$X_A = \frac{\text{number of moles of A}}{\text{total number of moles of all components}}$$

D. Molality

1. Symbol for molality: M
Do not confuse the symbol for molality (*m*) with the symbol for mass (*m*) that we used earlier. The symbol for molality is always in italics.

2. Formula for determining molality:

$$M = \frac{\text{moles solute}}{\text{kg solvent}}$$

3. Exercises

- a. A solution contains sucrose, $C_{12}H_{22}O_{11}$, in water. The solution has a total mass of 738 g and the mass percent of sucrose in the solution is 38.4%. Calculate the molality of the solution.

Solution: To calculate the molality of the solution, you need to know two things: the mass of the solute and the mass of the solvent. You can calculate the mass of the solute, sucrose, by using the information on the mass percent of the solution.

$$38.4 = \frac{\text{mass of sucrose}}{738 \text{ g solution}} \times 100$$

Solving for the mass of sucrose, we get

$$m_{\text{sucrose}} = \frac{38.4 \times 738}{100} = 283 \text{ g}$$

Since the solution consists of water and sucrose, the mass of water is

$$738 \text{ g} - 283 \text{ g} = 455 \text{ g}$$

Since molality is moles of solute/kilogram of solvent, we need to convert grams of solute to moles of solute, and grams of solvent to kilograms of solvent. Starting with the solute, we need the molar mass of sucrose if we are to calculate moles from mass. The molar mass of sucrose is

$$12(12.01) + 22(1.01) + 11(16.0) = 342 \text{ g}$$

Using this molar mass, we now convert to moles.

$$283 \text{ g sucrose} \times \frac{1 \text{ mol sucrose}}{342 \text{ g sucrose}} = 0.827 \text{ mol}$$

The unit for the solvent has to be changed from grams to kilograms.

$$455 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 0.455 \text{ kg}$$

The molality is therefore

$$m = \frac{0.827 \text{ mol solute}}{0.455 \text{ kg}} = 1.82$$

- b. A 0.633 m solution of vitamin B₆ (pyridoxine) in water is prepared by adding 53.5 g of vitamin B₆ to 5.00 × 10² g of water. Calculate the molar mass of vitamin B₆. (E3)

$$m = \frac{\text{moles solute}}{\text{kg solvent}} = \frac{\text{moles B}_6}{\text{kg H}_2\text{O}} = \frac{\frac{\text{mass Vit B}_6}{\text{MM Vit B}_6}}{\text{kg H}_2\text{O}}$$

$$0.633 \text{ m} = \frac{\text{moles B}_6}{.5 \text{ kg H}_2\text{O}}$$

$$\text{MM} = \frac{\text{mass Vit B}_6}{m (\text{kg H}_2\text{O})}$$

- or -

$$\text{moles B}_6 = 0.3167 \text{ moles B}_6$$

$$\text{MM} = \frac{53.5 \text{ g}}{(.633)(.5)}$$

~~MM~~
$$\text{MM} = \frac{\text{grams B}_6}{\text{moles B}_6} = \frac{53.5 \text{ g}}{.3167 \text{ mol}}$$

$$\text{MM} = 169.0 \text{ g/mol}$$

$$\text{MM}_{\text{B}_6} = 169.0 \text{ g/mol}$$

E. Molarity

1. Symbol for molarity: M

2. Formula for molarity:

$$M = \frac{\text{moles solute}}{\text{L of solution}}$$

3. What a given molarity can tell you

- a. The number of moles of solute in a liter of solution

Example: A 0.250 M solution of Na₂SO₄ has 0.250 mol of Na₂SO₄ in one liter of solution. Remember solution is not the same as solvent. A solution that is 0.250 M does *not* mean 0.250 mol of Na₂SO₄ in one liter of water.

- b. The moles of solute in a given volume of solution

You can use the molarity of a solution as a conversion factor, with which you can convert a given volume of solution to the number of moles of solute in that solution.

Example: How many moles of Na_2SO_4 are there in 0.300 L of solution that is 0.250 M?

$$0.250 \text{ M means } \frac{0.250 \text{ mol Na}_2\text{SO}_4}{1 \text{ L solution}}$$

Thus,

$$0.300 \text{ L} \times \frac{0.250 \text{ mol Na}_2\text{SO}_4}{1 \text{ L}} = 0.0750 \text{ mol}$$

- c. The volume of solution that is necessary, given the number of moles of solute and the molarity of the solution

Example: What is the volume of a 0.250 M solution that has 0.700 mol of Na_2SO_4 as solute?

Again, we use the conversion factor

$$\frac{0.250 \text{ mol Na}_2\text{SO}_4}{1 \text{ L solution}}$$

to convert 0.700 mol of solute to a volume of solution. Thus,

$$0.700 \text{ mol} \times \frac{1 \text{ L solution}}{0.250 \text{ mol Na}_2\text{SO}_4} = 2.80 \text{ L}$$

- d. The number of grams of solute in a liter of solution

Example: How many grams of Na_2SO_4 are in a liter of a solution that is 0.250 M?

Here you have to use the conversion factor

$$\frac{142 \text{ g Na}_2\text{SO}_4}{1 \text{ mol Na}_2\text{SO}_4}$$

Thus, since 0.250 M means 0.250 mol $\text{Na}_2\text{SO}_4/\text{L}$ of solution, we get

$$\frac{0.250 \text{ mol Na}_2\text{SO}_4}{\text{L solution}} \times \frac{142 \text{ g Na}_2\text{SO}_4}{1 \text{ mol Na}_2\text{SO}_4} = \frac{35.5 \text{ g Na}_2\text{SO}_4}{\text{L solution}}$$

- e. The number of grams of solute in a given volume of solution

Example: How many grams of Na_2SO_4 are in 0.800 L of a 0.250 M solution?

Again the conversion factors are:

$$\frac{142 \text{ g Na}_2\text{SO}_4}{1 \text{ mol Na}_2\text{SO}_4} \quad \text{and} \quad \frac{0.250 \text{ mol Na}_2\text{SO}_4}{\text{L solution}}$$

Here 0.800 L has to be converted to grams of Na_2SO_4 . So

$$0.800 \text{ L} \times \frac{0.250 \text{ mol Na}_2\text{SO}_4}{\text{L solution}} \times \frac{142 \text{ g Na}_2\text{SO}_4}{1 \text{ mol Na}_2\text{SO}_4} = 28.4 \text{ g Na}_2\text{SO}_4$$

f. The volume of a given molarity required for a given mass of solute

Example: What is the volume of a solution that has 157 g of Na_2SO_4 if the solution is 0.250 M?

The conversion factors again are:

$$\frac{142 \text{ g Na}_2\text{SO}_4}{1 \text{ mol Na}_2\text{SO}_4} \quad \text{and} \quad \frac{0.250 \text{ mol Na}_2\text{SO}_4}{\text{L solution}}$$

This time we have to change 157 g Na_2SO_4 to liters of solution. Thus,

$$157 \text{ g Na}_2\text{SO}_4 \times \frac{1 \text{ mol Na}_2\text{SO}_4}{142 \text{ g Na}_2\text{SO}_4} \times \frac{\text{L solution}}{0.250 \text{ mol Na}_2\text{SO}_4} = 4.42 \text{ L solution}$$

4. Exercises

a. Calculate the molarity of a solution prepared by adding enough water to 73.2 g of ammonium nitrate to make 0.835 L of solution. (E4)

$$M = \frac{\text{moles}}{\text{L}} = \frac{73.2 \text{ g NH}_4\text{NO}_3 \left(\frac{1 \text{ mol}}{80 \text{ g}} \right)}{0.835 \text{ L}} = \boxed{1.10 \text{ M NH}_4\text{NO}_3}$$

b. How many moles of calcium chloride do you need in 356 mL of solution to make a 0.125 M solution? (E5)

$$\text{moles} = M \times L = 0.125 \frac{\text{mol}}{\text{L}} \times 0.356 \text{ L} = \boxed{0.0445 \text{ mol CaCl}_2}$$

c. How many grams of potassium permanganate are required in 565 mL of solution to obtain a 0.380 M solution? (E6)

$$565 \text{ mL} \left(\frac{1 \text{ L}}{1000 \text{ mL}} \right) \left(\frac{0.380 \text{ mol}}{\text{L}} \right) \left(\frac{158.04 \text{ g}}{1 \text{ mol KMnO}_4} \right) = \boxed{33.9 \text{ g KMnO}_4}$$

d. What volume of solution is required for a 1.25 M solution of $\text{Pb}(\text{NO}_3)_2$ that contains 58.5 g of solute? (E7)

$$58.5 \text{ g Pb}(\text{NO}_3)_2 \left(\frac{1 \text{ mol Pb}(\text{NO}_3)_2}{331.2 \text{ g}} \right) \left(\frac{1 \text{ L Pb}(\text{NO}_3)_2}{1.25 \text{ mol}} \right) \left(\frac{1000 \text{ mL}}{1 \text{ L}} \right) = \boxed{141 \text{ mL solution}}$$

e. How many moles of phosphate and ammonium ions are there in 0.250 L of a 0.150 M solution of ammonium phosphate? (E8)

$$0.250 \text{ L} \left(\frac{0.150 \text{ mol}}{\text{L}} \right) = 0.0375 \text{ mol } (\text{NH}_4)_3\text{PO}_4 \rightarrow \begin{cases} \left(\frac{3 \text{ mol NH}_4^+ \text{ ions}}{1 \text{ mol } (\text{NH}_4)_3\text{PO}_4} \right) = 0.1125 \text{ mol NH}_4^+ \text{ ions} \\ \left(\frac{1 \text{ mol PO}_4^{3-}}{1 \text{ mol } (\text{NH}_4)_3\text{PO}_4} \right) = 0.0375 \text{ mol PO}_4^{3-} \text{ ions} \end{cases}$$

Caution: It can be very confusing to do molarity problems using ratios and proportions. The factor conversion method is simpler and clearer.

5. Preparing solutions by dilution
 a. Formula used for dilution problems:

$$M_1 V_1 = M_2 V_2 \quad \text{or} \quad M_c V_c = M_d V_d$$

C = concentrated
 D = Diluted

b. Exercises

- (1) How would you prepare 100.0 mL of a 0.200 M solution of NaOH from a bottle that is labeled 0.4871 M?

Solution: The concentrated solution is the one that is on hand. In this case it is the one labeled 0.4871 M. The dilute solution is the one that has to be prepared. In this case it is 0.200 M. What you want to find is the volume of concentrated solution to take so that you can add enough water to make 100.0 mL of a 0.200 M solution. Thus, V_c is unknown, M_c is 0.4871 M, V_d is 100.0 mL, and M_d is 0.200 M. Using the formula, we get

$$V_c = \frac{M_d \times V_d}{M_c} = \frac{0.200 \text{ M} \times 100.0 \text{ mL}}{0.4871 \text{ M}} = 41.1 \text{ mL}$$

You would prepare the final solution by taking 41.1 mL of the initial solution and adding enough water to make the final volume 100.0 mL. Note that in dilution problems, the volumes of the solutions do not have to be in liters. You just have to be consistent. In calculating molarity for other problems, however, you must always have the volume in liters.

- (2) Calculate the molarity of a solution prepared by taking 25.00 mL of a 0.400 M solution of HNO_3 and adding enough water to make 75.00 mL of solution. (E9)

$$\begin{aligned} M_c &= 0.4 \text{ M} \\ V_c &= 25 \text{ mL} \\ M_d &= ? \\ V_d &= 75 \text{ mL} \end{aligned}$$

$$M_c V_c = M_d V_d$$

$$(0.4 \text{ M})(25 \text{ mL}) = M_d (75 \text{ mL})$$

$$M_d = 0.133 \text{ M}$$

F. Relating concentration units to each other

1. If you examine all concentration units closely, you see that the numerator of each unit is either in grams of solute or in moles of solute. The denominator in each unit is the one that varies. When you want to convert from one concentration unit to another, it is wise to separate the numerator from the denominator and work with them as separate entities. After you have gotten the required units for both numerator and denominator, put them together.
2. Given a concentration unit, it is also a good idea to arbitrarily choose one unit of the denominator. For example, if you were asked to calculate the molarity of a 2.50 molal solution, then choose one kilogram as the unit for the solvent since that is the denominator unit of the given molality. The numerator is of course 2.50 moles of solute, which is also the numerator for molarity.
3. It is also important to know the density in most of these interconversions.
4. You will find that organizing your data by filling in a prepared table is the easiest way to do these interconversions. You may not need all the entries, but you will find that the data is much more accessible arranged this way. A blank table should look like this:

	mass (g)	moles	volume (L)
solute			X
solvent			X
solution			

We put X's for the volumes of solute and solvent because we will never need those entries. The use of the table will be shown in the following exercise.

5. Exercises

- a. What is the molarity of an aqueous solution of HCl that is 35.0% by mass and whose density is 1.19 g/mL?

Solution: Since we are given mass percent (mass of solute/100 g of solution), we arbitrarily choose to work with 100.0 grams of solution. Doing that, we can deduce that there are 35.0 g of solute and 65.0 g of solvent. Your table should now look like this:

	mass (g)	moles	volume (L)
solute	35.0		X
solvent	65.0		X
solution	100.0		

Knowing that the solute is HCl ($M = 36.45$ g/mol) and that the solvent is water, ($M = 18.02$ g/mol), we can fill in the table with the results of the following calculations.

$$35.0 \text{ g HCl} \times \frac{1 \text{ mol}}{36.45 \text{ g}} = 0.960 \text{ mol HCl}$$

$$65.0 \text{ g H}_2\text{O} \times \frac{1 \text{ mol}}{18.02 \text{ g}} = 3.61 \text{ mol H}_2\text{O}$$

$$\text{mol solution} = 0.960 + 3.61 = 4.57$$

	mass (g)	moles	volume (L)
solute	35.0	0.960	X
solvent	65.0	3.61	X
solution	100.0	4.57	

CONTINUED →

	mass (g)	moles	volume (L)
solute	35.0	0.960	X
solvent	65.0	3.61	X
solution	100.0	4.57	0.0840

Note that we did all this without worrying about what was asked for. Now, no matter what concentration unit is asked for, you can easily calculate it, provided that you know the definition for that concentration unit. In this case, the unit asked for is molarity. Thus using the definition for molarity and picking out the appropriate data from the table, we get

$$M = \frac{\text{mol solute}}{\text{L solution}} = \frac{0.960 \text{ mol HCl}}{0.0840 \text{ L solution}} = 11.4$$

We could, however, just as easily have solved for molality or mole fraction of HCl.

- b. How would you prepare 5.00 L of 2.00 M H_2SO_4 from a bottle labeled: H_2SO_4 — 98.0% by mass; density — 1.84 g/mL? (E10)

① Fill in chart:

	mass (g)	moles	Volume (L)
Solute (H_2SO_4)	98 g	$98 \div 98 =$ 1 mole H_2SO_4	X
Solvent (H_2O)	2 g	$2 \div 18 =$.111 mol H_2O	X
Solution	100 g	1.111 total moles	$D = \frac{M}{V}$ $V = \frac{M}{D} = \frac{100g}{1.84g/mL} = 54.3 \text{ mL}$ <u>.0543 L</u>

② Find Molarity of Bottle: $M = \frac{\text{mol}}{\text{L}} = \frac{1 \text{ mol } \text{H}_2\text{SO}_4}{.0543 \text{ L}} = 18.4 \text{ M}$

③ Use Dilution = $M_c V_c = M_d V_d$

$$M_c = 18.4 \text{ M}$$

$$M_d = 2 \text{ M}$$

$$V_d = 5 \text{ L}$$

$$V_c = ?$$

$$(18.4 \text{ M})(V_c) = (2 \text{ M})(5 \text{ L})$$

$$V_c = .543 \text{ L} = 543 \text{ mL } \text{H}_2\text{SO}_4$$

and add $4.457 \text{ L of } \text{H}_2\text{O}$

- c. Calculate the molality of a 2.500 M solution of sulfuric acid. Its density is 1.200 g/mL.

Solution: We choose 1.000 L of solution, which therefore means that we have 2.500 moles of solute (from the definition of molarity). We list these as our initial entries in the table.

	mass (g)	moles	volume (L)
solute		2.500	X
solvent			X
solution			1.000

There are many ways in which you can now proceed. We choose to determine the mass of the solution by using the density of the solution that is given.

$$1000 \text{ mL solution} \times \frac{1.200 \text{ g solution}}{1 \text{ mL solution}} = 1.200 \times 10^3 \text{ g solution}$$

Our table now looks like this:

	mass (g)	moles	volume (L)
solute		2.500	X
solvent			X
solution	1.200×10^3		1.000

We now determine the mass of the solute using the molar mass of H_2SO_4 .

$$2.500 \text{ mol H}_2\text{SO}_4 \times \frac{98.08 \text{ g}}{1 \text{ mol}} = 245.2 \text{ g H}_2\text{SO}_4$$

We enter that value in our table.

	mass (g)	moles	volume (L)
solute	245.2	2.500	X
solvent			X
solution	1.200×10^3		1.000

Now, we can determine the mass of solvent ($1.200 \times 10^3 - 245.2$) and the number of moles of solvent, water. Finally, we determine the moles of solution by adding moles of solute and moles of solvent. The completed table now looks like this.

	mass (g)	moles	volume (L)
solute	245.2	2.500	X
solvent	955	53.0	X
solution	1.200×10^3	55.5	1.000

Since the problem asks for the molality of the solution, we pick the moles of solute from the table (2.500 mol) and the mass of solvent in kg (0.955). Thus

$$m = \frac{2.500 \text{ mol solute}}{0.955 \text{ kg solvent}} = 2.62$$

- d. Calculate the mol fraction of H_2SO_4 , its mass percent, and parts per million in the solution in c. (E11) $2.500 M H_2SO_4$ $d = 1.20 g/mL$

	mass (g)	moles	volume (L)
H_2SO_4 solute	③ $2.5 \text{ mol} \times 98.9 \text{ g/mol} = 245 \text{ g } H_2SO_4$	$2.500 \frac{\text{mol}}{L} \times 1 L = 2.50 \text{ mol } H_2SO_4$	X
H_2O solvent	⑤ $1200 \text{ g} - 245 \text{ g} = 955 \text{ g } H_2O$	⑥ $955 \text{ g} \div 18.9 \text{ g/mol} = 53.1 \text{ mol } H_2O$	X
sol'n	④ $1000 \text{ mL} \times 1.20 \text{ g/mL} = 1200 \text{ g}$	⑦ $53.1 + 2.50 = 55.6 \text{ mol sol'n}$	① 1 L

$$\% \text{ mass} = \frac{\text{mass solute}}{\text{mass sol'n}} \times 100 = \frac{245 \text{ g}}{1200 \text{ g}} \times 100 = \underline{20.4\% H_2SO_4}$$

$$\text{ppm} = \frac{\text{mass solute}}{\text{mass sol'n}} \times 10^6 = \frac{245}{1200} \times 10^6 = 204167 \text{ ppm} = \underline{2.04 \times 10^5 \text{ ppm}}$$

$$X_{H_2SO_4} = \frac{\text{mol } H_2SO_4}{\text{total moles}} = \frac{2.50 \text{ mol}}{55.6} = \underline{.0450}$$

II. Principles of Solubility

A. Solute-solvent interactions

- Solubility is dependent on both the nature of the solute and the solvent.
- Electrical forces that determine the extent to which ionic compounds dissolve in water are

a. The force between ions in the crystal and polar water molecules

b. The force between opposing charged ions in the crystal.

- "Like dissolves like" principle

a. Ionic substances tend to dissolve in polar solvents.

b. Nonpolar or slightly polar solutes are most soluble in solvents that are

(polarity) non-polar "like likes like!"